Exact Pareto-Optimal Coordination of Two Translating Polygonal Robots on an Acyclic Roadmap

Hamidreza Chitsaz, Jason M. O'Kane and Steven M. LaValle

{chitsaz, jokane, lavalle}@cs.uiuc.edu.

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

Introduction

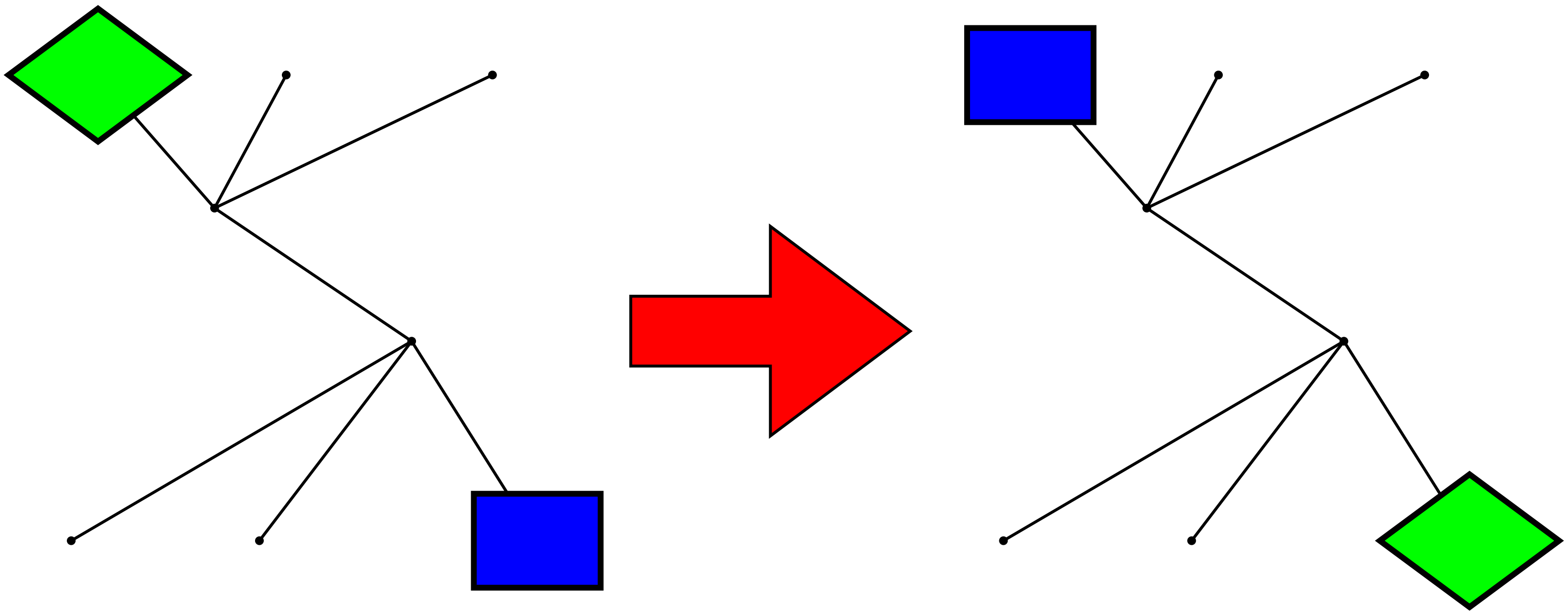
We want to study coordination strategies for robots in a shared workspace. We allow to individual robots to have separate performance measures.

Problem: Find collision-free motion strategies that are optimal in a multi-objective sense.

Applications:

- AGVs in a factory setting.
- General multiple robot coordination.

Introduction



Overview

- Related Work
- Problem Statement
- Robots on Fixed Paths
- Generalization to Roadmaps
- Examples

Related Work

Centralized Methods

Schwartz and Sharir, 1983

Ardema and Skowronski, 1991

Barraquand and Latombe, 1991

Decentralized (“Fixed Path”) Methods

Erdmann and Lozano-Perez, 1986

Akella and Hutchinson, 2002

Siméon, Leroy and Laumond 2002

Peng and Akella, 2003

Roadmap Methods

LaValle and Hutchinson, 1998

Problem Statement

Two polygonal robots $\mathcal{R}_1$ and $\mathcal{R}_2$ translating in the plane.

- Robots move on roadmaps $\mathcal{G}_1$ and $\mathcal{G}_2$ of piecewise-linear paths.
- Initial and goal configurations $X_i^{init}, X_i^{goal} \in \mathcal{G}_i$.
- Allow instantaneous changes in speed.

Objective: Find a continuous collision-free path

$$C : [0,1] \rightarrow \mathcal{G}_1 \times \mathcal{G}_2$$

from (X_1^{init}, X_2^{init}) to (X_1^{goal}, X_2^{goal}).

Optimality

For a given coordination, each robot has a cost function:

$$J = (J_1, J_2)$$

One approach is to choose a scalarization function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ and optimize $f(J)$.

- Scalarizing may omit interesting solutions.
- Priorities may change across multiple queries.
- Better to find a small set of good candidate solutions.

Pareto Optimality

Rather than choosing any particular scalarization, we find the set of **Pareto optimal** solutions.

- Create equivalence classes of paths with identical costs.

$$C \sim C' := \quad J(C) = J(C')$$

- Define a partial order on equivalence classes:

$$[C] \leq [C'] := J_1(C) \leq J_1(C') \wedge J_2(C) \leq J_2(\mathcal{C}')$$

- Pareto optima are the minima in this partial order.

Coordination Space

O'Donnell and Lozano-Perez, 1989

We want to find a path through the *coordination space* $\mathcal{G}_1 \times \mathcal{G}_2$.

- Obstacle regions where $\mathcal{R}_1$ collides with $\mathcal{R}_2$.

The slope of this curve determines the velocity of each robot.

- Slope ≥ 1: $\mathcal{R}_2$ at full speed
- Slope ≤ 1: $\mathcal{R}_1$ at full speed.

Time to execute a segment is its L^∞ length.

Optima for Fixed Paths

Lemma: Every Pareto-optimal coordination class contains a coordination composed of segments of the visibility graph of the obstacle set, plus possibly a “full speed completion.”

Proof Ideas:

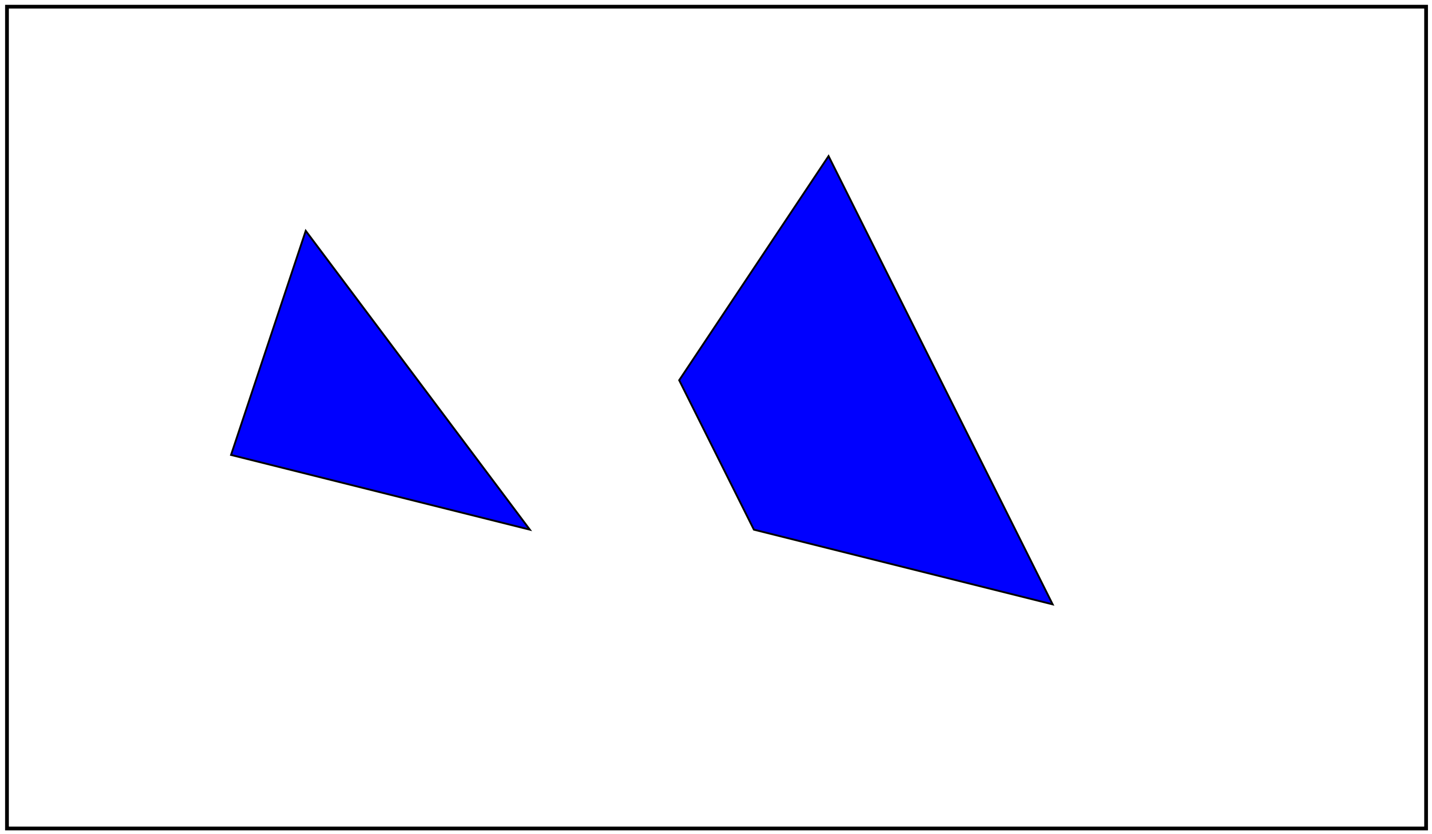
- Given any path, “shorten” it until it’s constrained by obstacle vertices.
- After moving past the last obstruction, both robots should move at full speed to their goals.

Optima for Fixed Paths

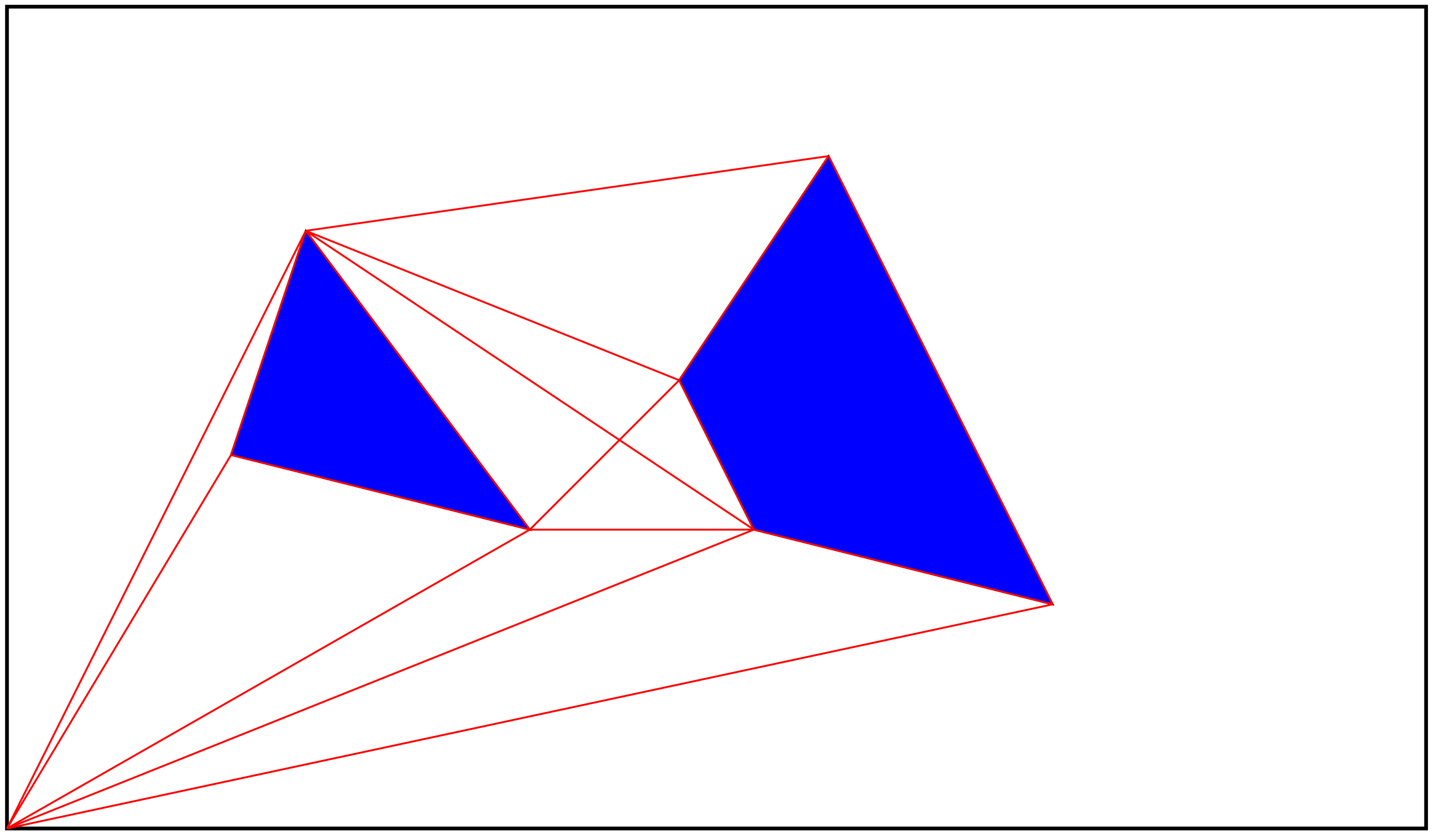
Algorithm:

- Compute the obstacle set.
- Find the visibility graph of obstacle set.
- Add a full-speed completion from each vertex for which this is possible.
- Use Dijkstra's algorithm to extract a set of candidate solutions.
  - Candidate = Shortest path to obstacle vertex + full-speed completion.
- Use direct comparisons to eliminate candidates that are not Pareto optima.

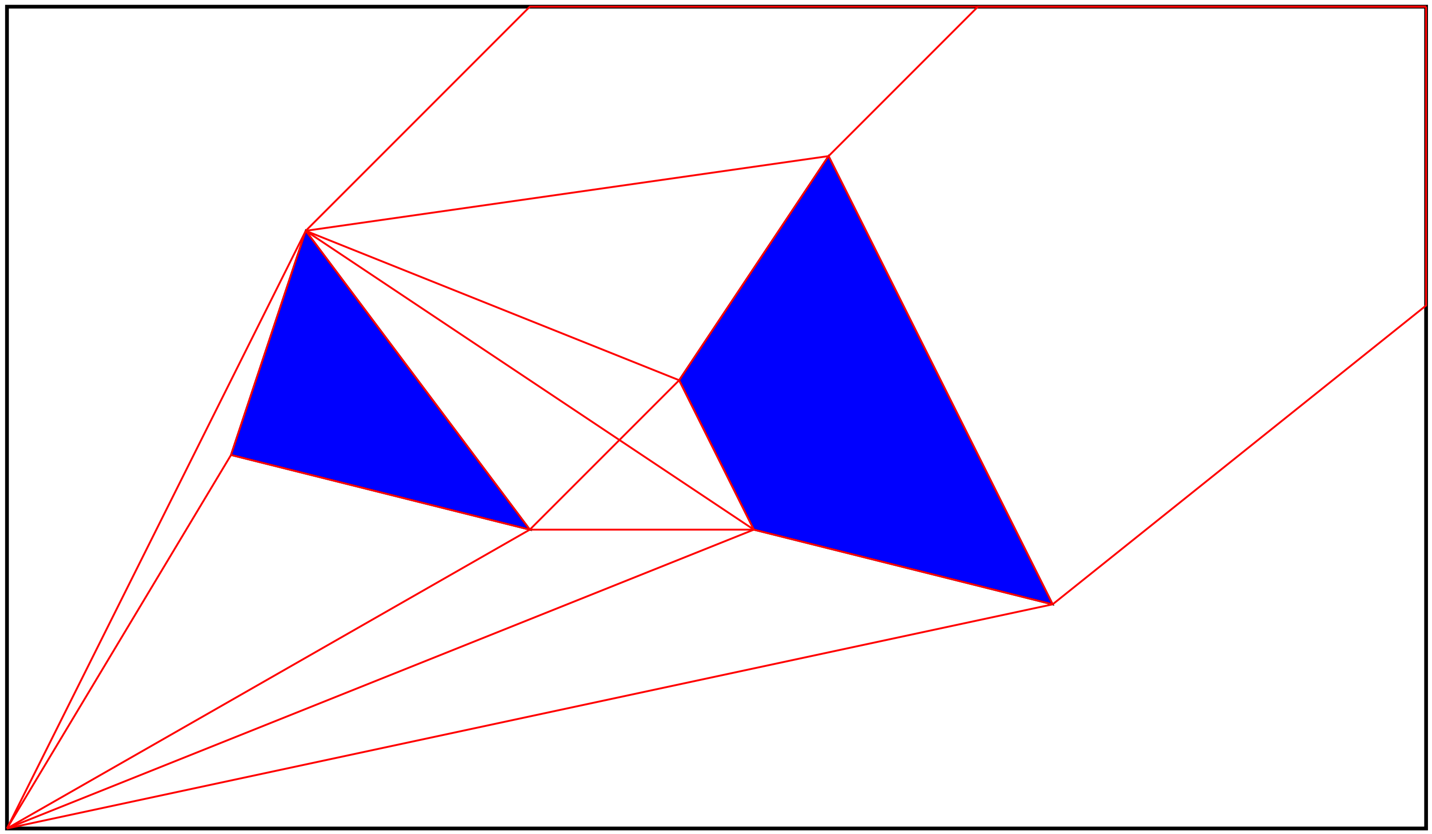
Fixed Path Example



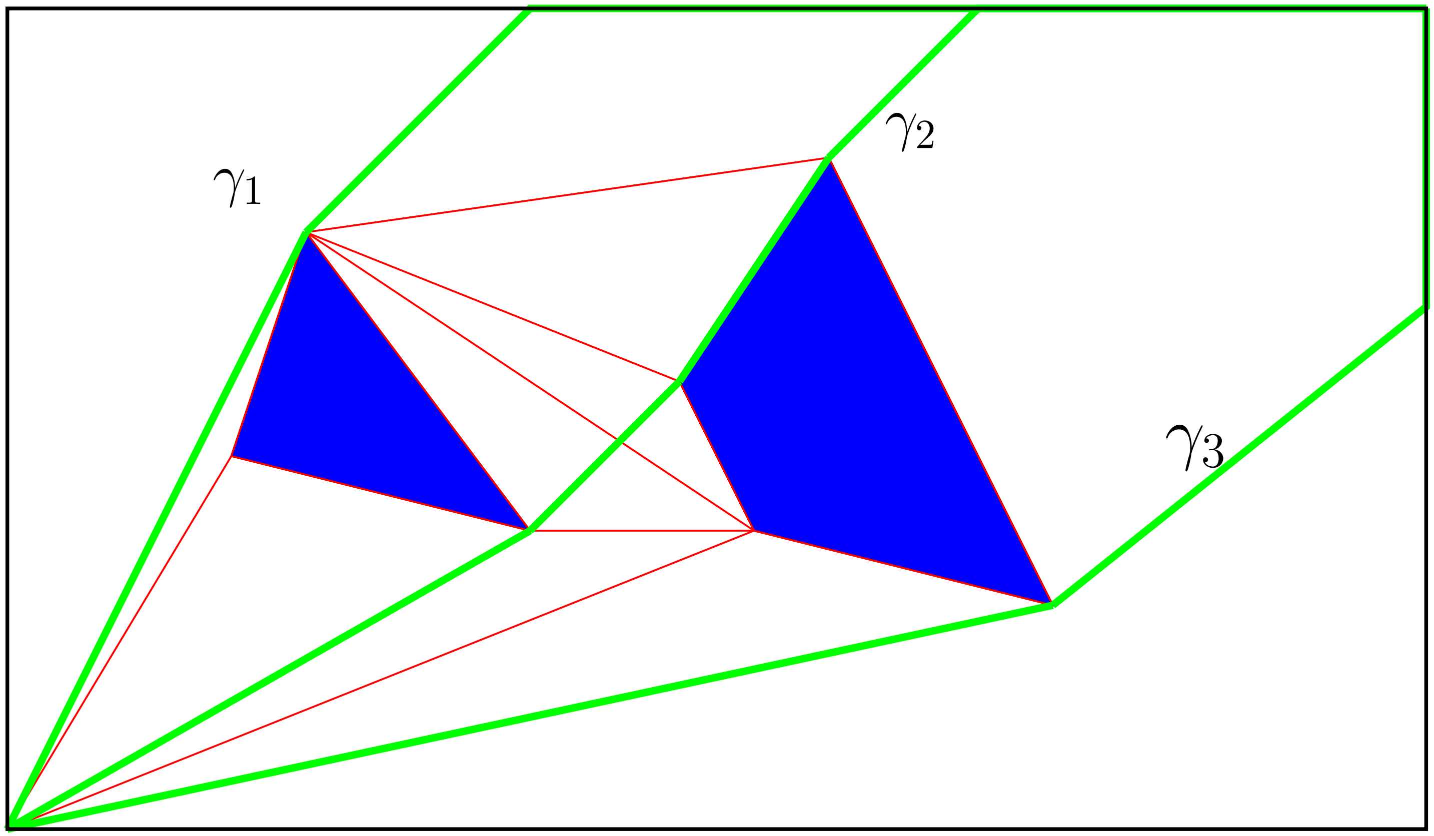
Fixed Path Example



Fixed Path Example

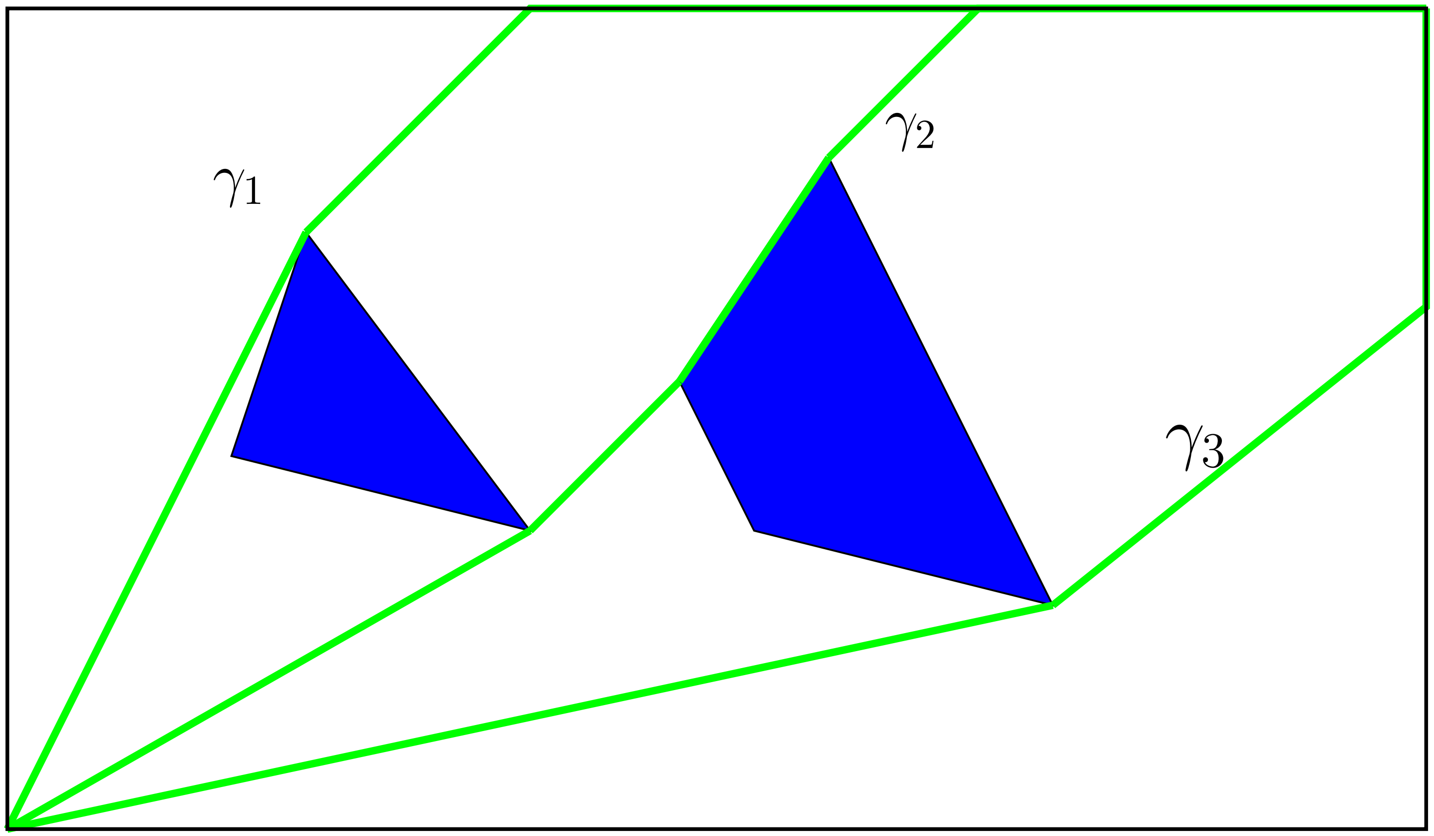


Fixed Path Example



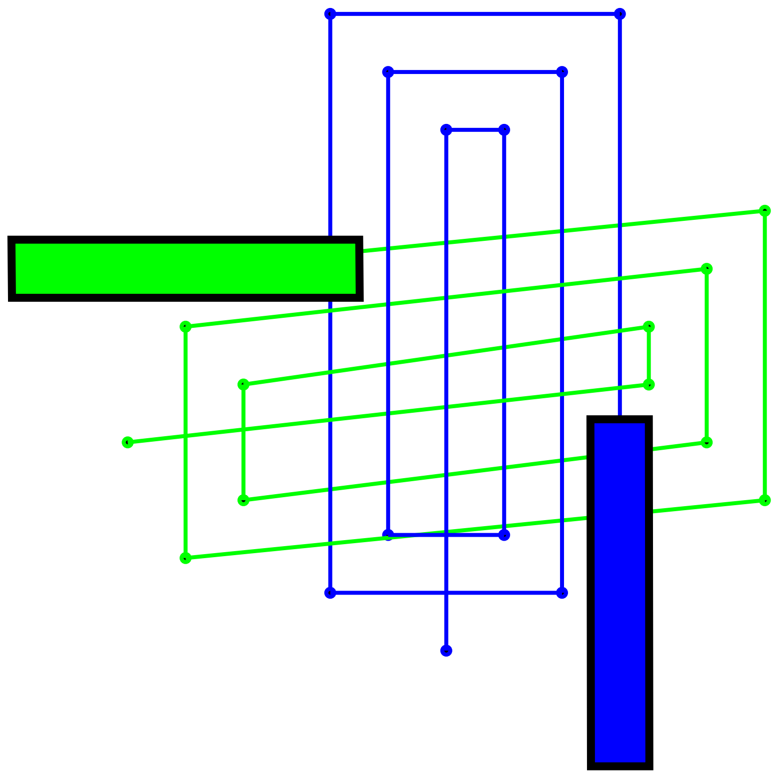
$$J(\gamma_1) = (23, 11)$$
$$J(\gamma_2) = (21, 15)$$
$$J(\gamma_3) = (19, 25)$$

Fixed Path Example



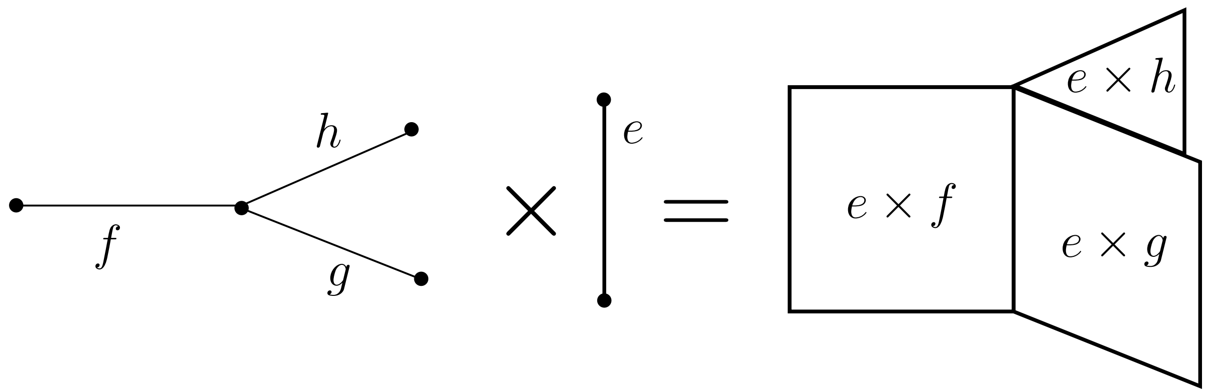
$$
\begin{aligned}
J(\gamma_1) &= (23, 11) \\
J(\gamma_2) &= (21, 15) \\
J(\gamma_3) &= (19, 25)
\end{aligned}
$$

Example



Acyclic Roadmaps

$\mathcal{G}_1 \times \mathcal{G}_1$ is a collection of 2-dimensional cells pasted together at their boundaries.



Same method works. Only need a technique to compute the visibility graph.

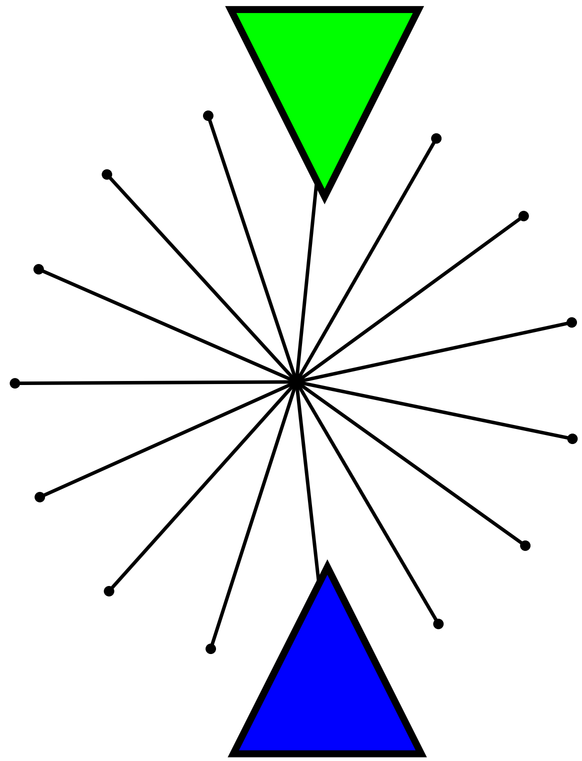
Visibility in $\mathcal{G}_1 \times \mathcal{G}_2$

Standard algorithm for $\mathbb{R}^2$ *(Lee, 1978):* Radial sweep about each vertex. Maintain a balanced tree of intersected segments. $O(n^2 \log n)$ time.

Our extension: Radial sweep in $\mathcal{G}_1 \times \mathcal{G}_2$. Maintain a separate balanced tree in each for each cell.

- A ray in $\mathcal{G}_1 \times \mathcal{G}_2$ passes through at most $2m$ cells, where m is the total number of edges.
- At most $2m$ binary tree operations to process each event.
- $O(mn^2 \log n)$ time.

Example



Conclusion

- Pareto optimality is an important solution concept for multiple robot coordination.
- Presented an $O(m^2 n \log n)$ time algorithm to compute all Pareto optima for problems with m edges in the roadmaps and n obstacle vertices.
- Future Work:
  - n robots. (with R. Ghrist)
  - Cyclic graphs.